

Comment on “Finite Size Corrections to the Radiation Reaction Force in Classical Electrodynamics”

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In Ref. [1] effective field theory methods have been employed to compute the equations of motion of a spherically symmetric charged shell of radius R , taking into account the radiation reaction force exerted by the shell’s own electromagnetic field up to $\mathcal{O}(R^2)$. The authors of Ref. [1] have stated that the known result for the self-force of the shell as can be found from Eq. (16.28) of the textbook of Jackson [2] (see also Chap. 4 in the review of Pearle [3])

$$\begin{aligned}\mathbf{F}_{\text{self}} &= \frac{e^2}{3R^2}(\mathbf{v}(t-2R) - \mathbf{v}(t)) \\ &= \frac{2}{3}e^2 \left(-\frac{1}{R}\ddot{\mathbf{x}} + \ddot{\mathbf{x}} - \frac{2}{3}R\mathbf{x}^{(4)} + \dots \right) \quad (1)\end{aligned}$$

is incorrect, in that the term linear in R should be absent. We claim that this conclusion of Ref. [1] is incorrect, and that the textbook result, Eq. (1) does hold.

First of all we note that in his monumental work [4] Nodvik has derived the equations of motion of an extended, rigid, spherically symmetric charge distribution of radius R taking into account its radiation reaction force (or self-force) up to $\mathcal{O}(R^2)$ in a manifestly relativistically covariant formalism. A first inspection of Ref. [4] shows that there are in fact terms linear in R in the equations of motion. In the special case of a charged shell, the four-vector of the self-force contains a term proportional to R , given as

$$F_{1,\text{sh}}^\mu = \frac{2}{3}e^2 R \left[2\dot{x}^\mu(\ddot{x}_\nu \ddot{x}^\nu) + \ddot{x}^\mu(\ddot{x}_\nu \ddot{x}^\nu) - \frac{2}{3}x^{(4)\mu} \right], \quad (2)$$

and the nonrelativistic limit of (2) yields immediately the textbook result Eq. (1).

Independently of Ref. [4] we have rederived the radiation reaction force of an extended, rigid, spherically symmetric charge distribution and we are in full agreement with Nodvik’s results, and in particular with Eq. (2).

Pearle has derived an integral representation of the self-force of an extended, rigid, spherically symmetric charge distribution, Eq. (9.26) of Ref. [3]. We have also performed a direct computation starting from Eq. (9.26) of Ref. [3] up to $\mathcal{O}(R^2)$ to check the correctness of our results. We have reproduced Eq. (2) and at the same time we could also confirm the correctness of the $\mathcal{O}(R^2)$ term computed in Ref. [1]. Since the results of Ref. [3] all fit in the rather general framework of Nodvik, it is only to be expected that the results should agree.

Galley et al. argue that the most general world-line action of an extended shell, Eq. (17) of Ref. [1] cannot contain $\mathcal{O}(R)$ terms, since they are excluded by Poincaré and gauge symmetries together with reparametrization invariance of the world-line action. This argument is correct for terms containing the gauge field, A_μ , however the effective action of an extended charged object does contain a term *not explicitly depending on A_μ* and being of $\mathcal{O}(R)$. The appearance of such a term is due to the relativistic kinematics of an extended rigid body as it is well exposed in Ref. [4], where also the effective action is given. From Eq. (7.73) of Ref. [4] one can obtain the $\mathcal{O}(R)$ term for a shell in the action, given as

$$S^{(1)} = -\frac{2}{9}Re^2 \int d\tau a^2(\tau). \quad (3)$$

As one can see $S^{(1)}$ does not depend explicitly on A_μ and it is consistent with all symmetry requirements.

In fact one can learn from Refs. [5, 6] that the radiated four momentum receives contributions only from the difference of the advanced and the retarded potentials, which is an even function of R . Therefore it is not surprising that Galley et al. could match their result (Eq. (22) of Ref. [1]) based on an incomplete effective action for the radiated power to the result of Ref. [7]. However, the total four-momentum of the self-field generated by the shell, contributing to the self-force in the equations of motion contains a “bound” part depending of both even and odd powers of the shell’s radius [8].

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